COMPUTATIONAL GEOMETRY
Homework Set # 1

Due (upload to Brightspace) by Thursday, September 8, 2022.

Relevant Lecture Modules: 1–8.

Required Reading: Devadoss-O'Rourke, Chapter 1 (sections 1.1-1.3); O'Rourke, Chapter 1 (sections 1.1-1.2).

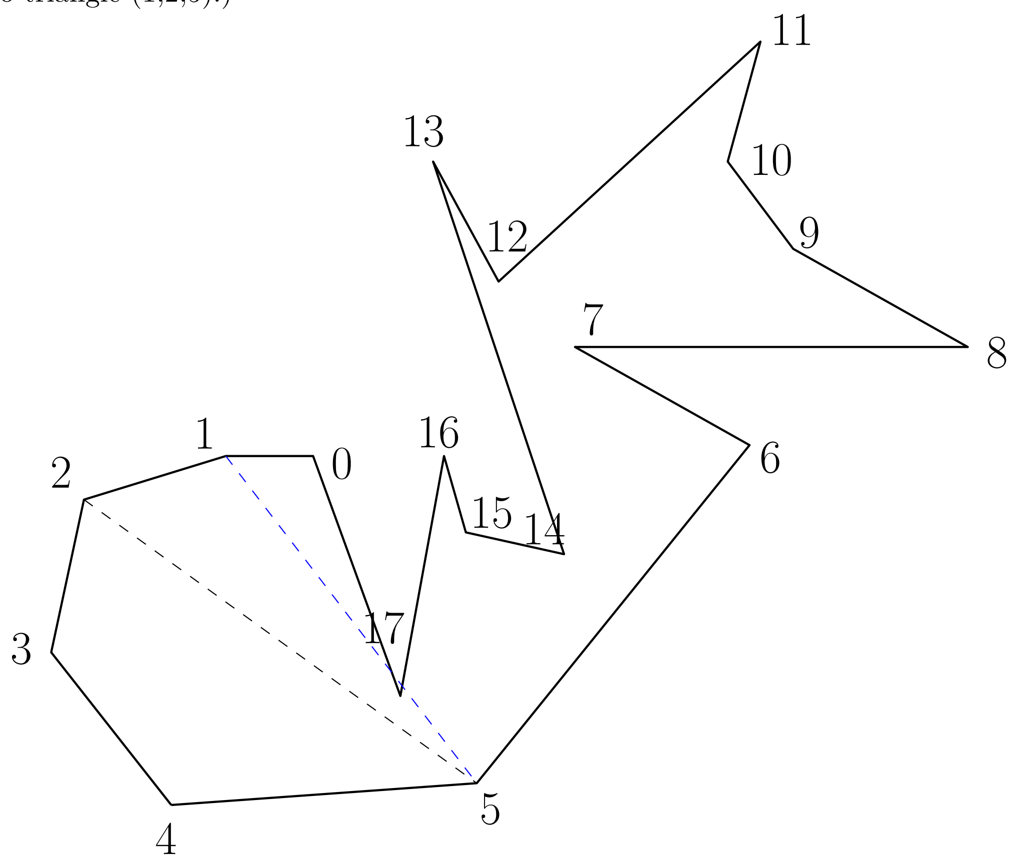
Optional Reading (relevant to the optional problems): Devadoss-O'Rourke, Chapter 1 (sections 1.4-1.5)

Reminder: In all of the exercises, be sure to give at least a brief explanation or justification for each claim that you make.

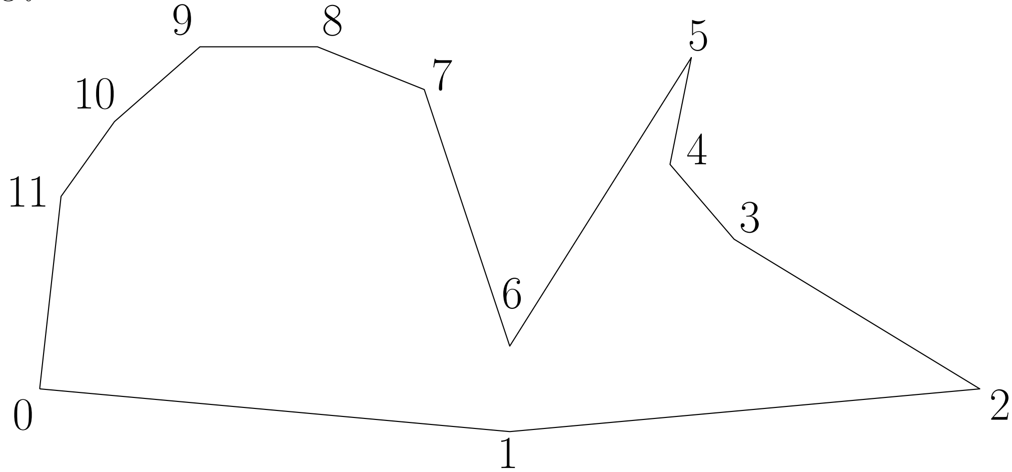
PROBLEMS TO TURN IN:

(1). [5 points] (Refer to Problem 1.16, Devadoss-O'Rourke.) (a). For the polygon below, find the number of distinct triangulations. Show your work, explaining your answer.

(I show (dashed) the diagonals (2,5) and (1,5), in order to make it clear that vertex 17 lies interior to triangle (1,2,5).)



(b). For the polygon below, find the number of distinct triangulations. Show your work, explaining your answer.



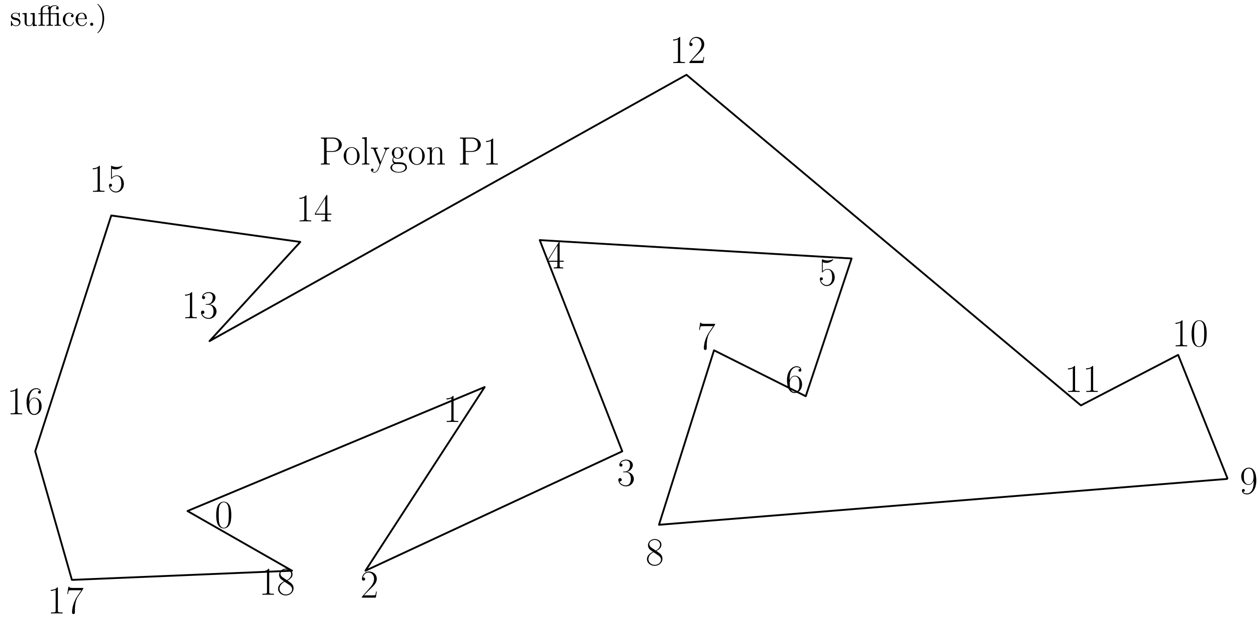
(2). [5 points] For the simple polygons P below, do the following:

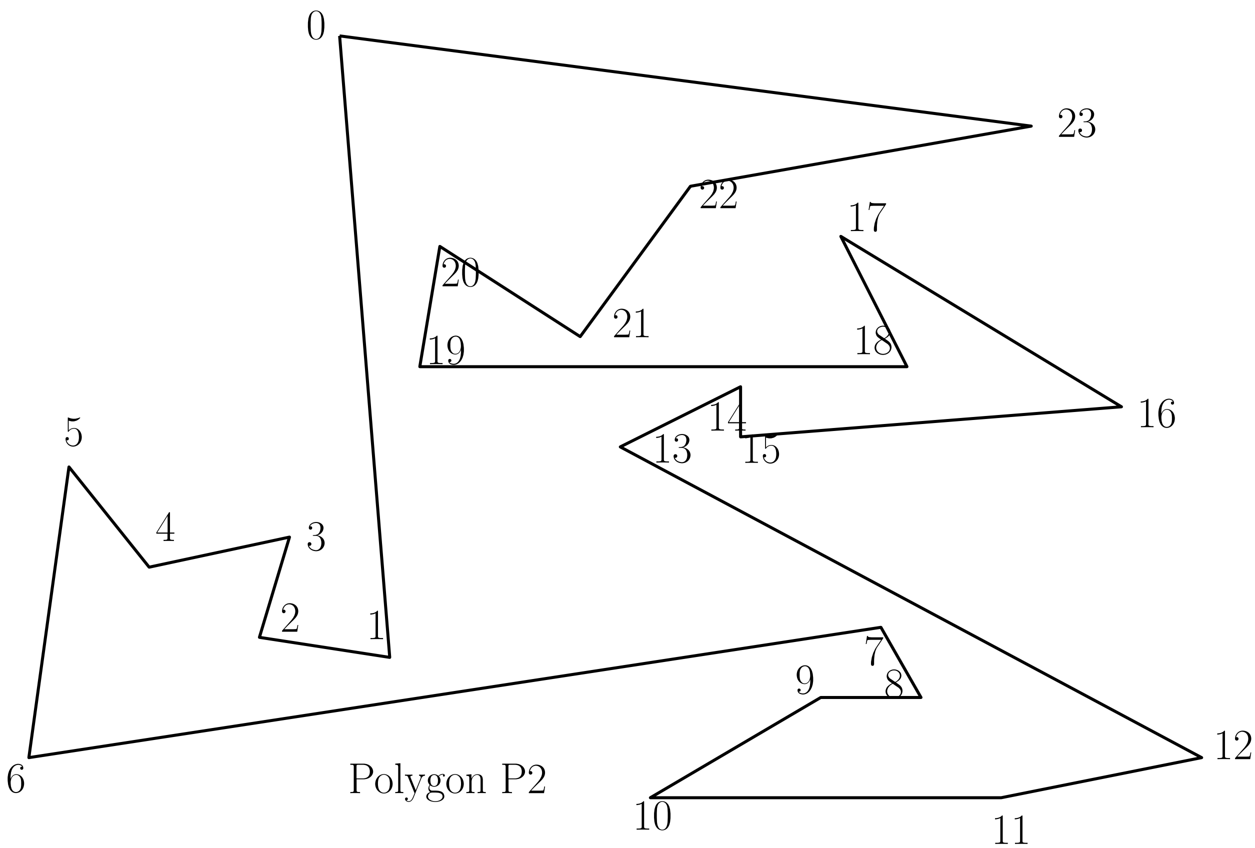
(a). Show a set of diagonals that yield a triangulation of P.

(b). Apply the method of Fisk's proof to obtain a set of at most $\lfloor n/3 \rfloor$ (vertex) guards. How many guards do you obtain? (Use the triangulation of P in your answer to part (a).)

(c). Obtain the *vertex* guard number for P; i.e., find the minimum number of vertex guards necessary to guard the polygon. Justify your answer! In particular, give an argument that fewer guards cannot suffice.

(d). Obtain the *point* guard number for P, allowing guards to be placed at *any* point (interior or boundary) of the polygon. Justify your answer! (Give an argument that fewer guards cannot suffice.)

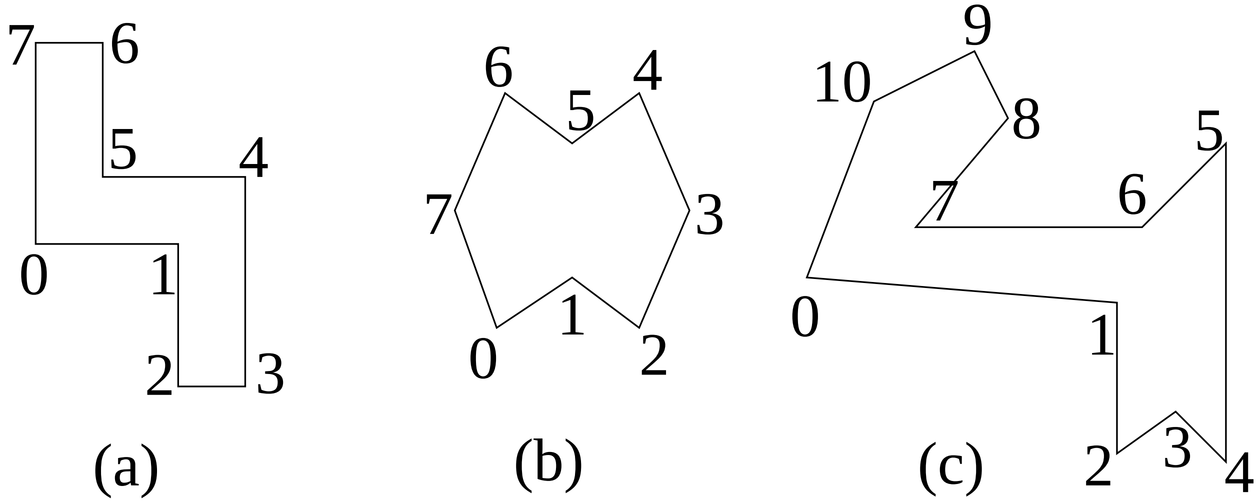




Polygon P2

ADDITIONAL PRACTICE PROBLEMS: TRY THEM, UNDERSTAND THEM, YOU ARE RESPONSIBLE FOR THEM

(3). (Refer to Problem 1.16, Devadoss-O'Rourke.) For each polygon below, find the number of distinct triangulations. Make sure to describe how you obtain your answer.



(4). (a). Problem 1.21, Devadoss-O'Rourke: For each $n > 3$, find a polygon with n vertices with exactly two triangulations.

In other words, find a *generic family* of examples of n-gons, each having exactly two triangulations, such that it is clear that your family includes arbitrarily large n-gons – e.g., we have seen the family of convex n-gons, Chvatal combs (which were defined for multiples of 3, $n = 3k$, but extend to values of n not divisible by 3), etc.

(b). Can you do the same for finding a polygon with n vertices (for each $n > 4$) with exactly **three** different triangulations?

(c). [Optional] Find a simple polygon P with n vertices, with n as small as possible such that P has exactly 4 different triangulations. Justify your answer (why P has exactly 4 different triangulations, and why there is no simple polygon with this property that has fewer vertices than the one you found).

(5). Give an example of a simple polygon P and a placement of some number of guards in P such that the guards see every one of the *vertices*, $V(P)$, of P, but there is at least one point on the boundary, ∂P, of P that is not seen by any of the guards.

Try to make your example as "small" as possible (having the fewest number of vertices in P). Optional: Can you argue that your example is the smallest possible?

(6). (a). Give an example of a simple polygon P and a set of 7 guards that cover it such that deletion of any one guard causes part of the gallery P to be unseen (i.e., the set of 7 guards is *minimal*), but the guard number, $g(P)$, for P is *less* than 7 ($g(P) < 7$).

(b). Now give an example of a simple polygon P, with $n = 15$ vertices, and a set G of 5 point guards that cover (see all of) P, such that the guards G form a minimal set (removing any one of them causes part of P not to be seen), but the optimal guard number is at most 2. (Make sure to justify! What exactly is the guard number, $g(P)$, of the polygon, and why?)

(c). Give an example of a simple polygon P such that the only way it is possible to obtain the maximum number, $w(P)$, of independent witness points (independent with respect to point guards) is to have at least one of the witness points be strictly interior to P (not on the boundary of P, and, in particular, not at vertices of P). Show an optimal set of $w(P)$ witness points, and also give the (point) guard number, $g(P)$ for your example.

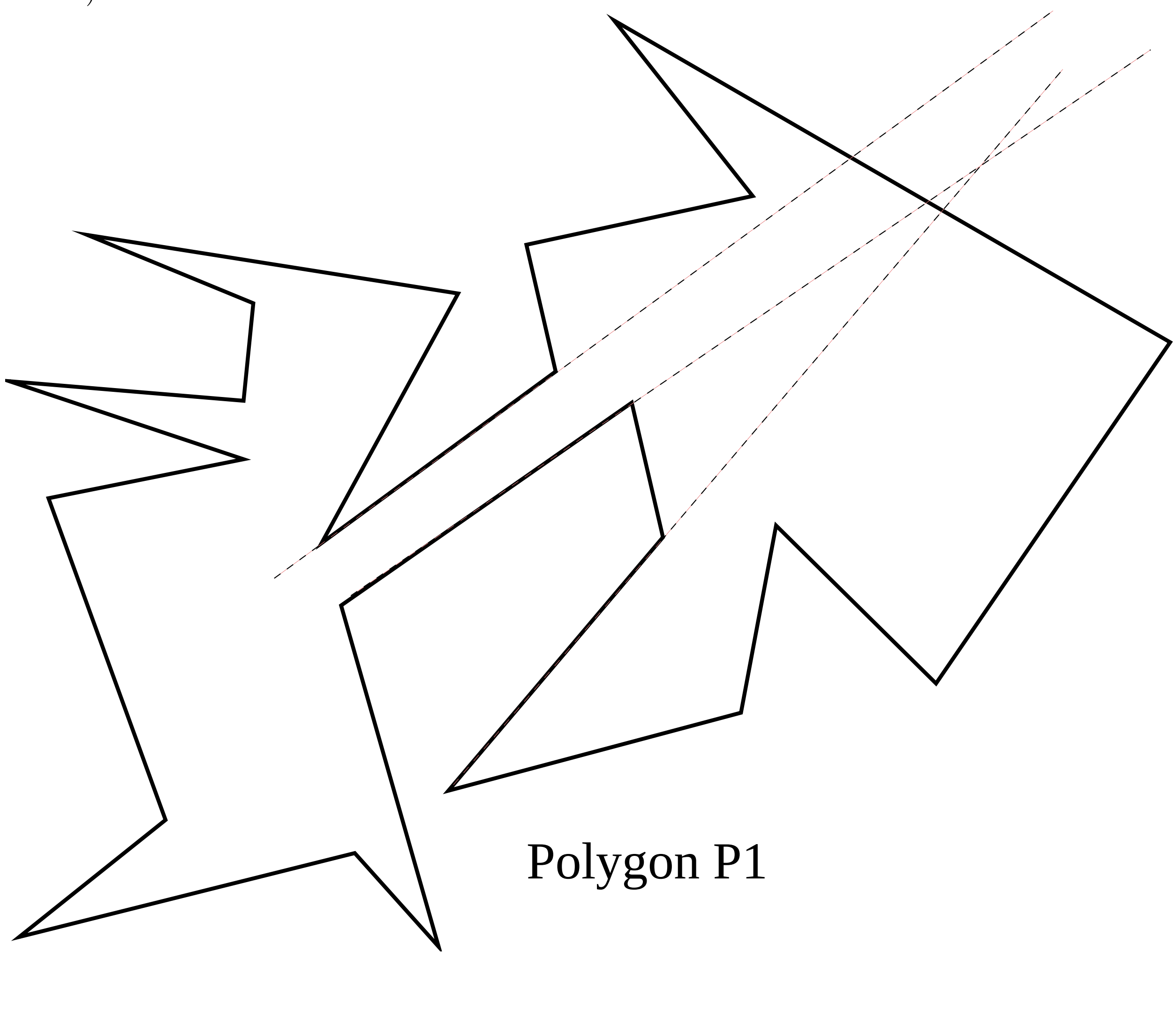
(7). For each of the simple polygons P below, do the following:

(a). Show a set of diagonals that yield a triangulation of P.

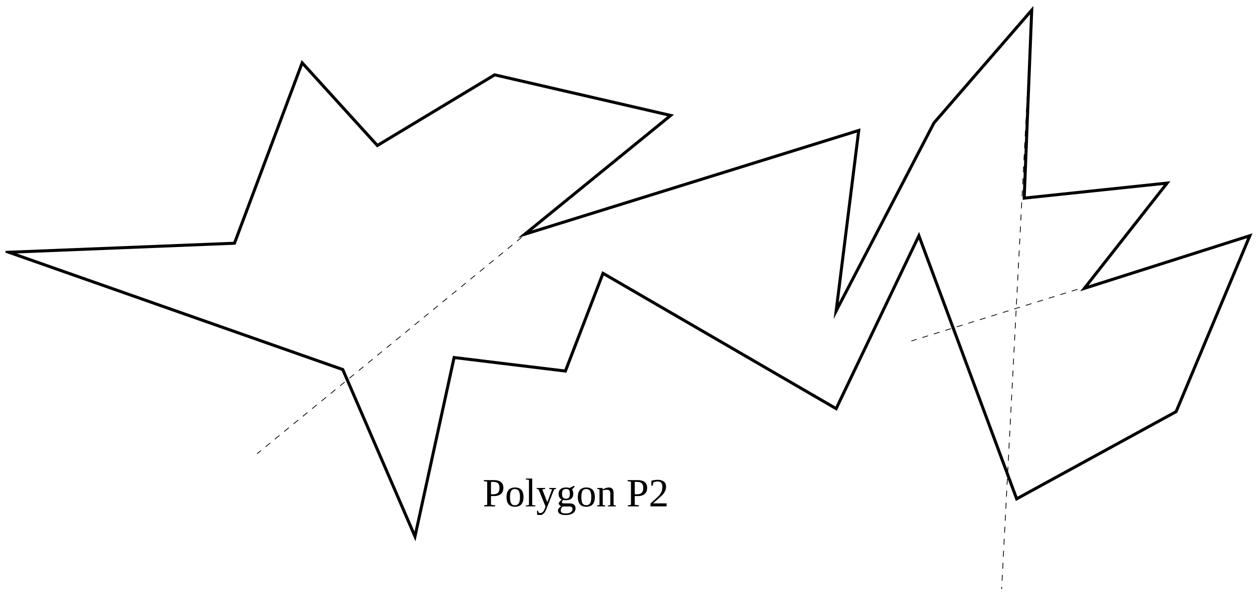
(b). Apply the method of Fisk's proof to obtain a set of at most $\lfloor n/3 \rfloor$ (vertex) guards. How many guards do you use?

(c). Obtain the *vertex* guard number for P; i.e., find the minimum number of vertex guards necessary to guard the polygon. Justify your answer! In particular, give an argument that fewer guards cannot suffice.

(d). Obtain the *point* guard number for P, allowing guards to be placed at *any* point (interior or boundary) of the polygon. Justify your answer! (Give an argument that fewer guards cannot suffice.)



Polygon P1



OPTIONAL IMPLEMENTATION: (adds up to 10 points to your overall hw score) Implement an easily executable code that allows the user to mouse in a simple polygon, and the code computes the number of different triangulations of the polygon. (Use the recursive computation (dynamic program) we discussed briefly in class.) Test your code for correctness on several examples (e.g., the ones from class, the hw, etc). You may use any language you wish, but the code should be as portable and easily usable as possible, ideally executable from a webpage, so that it can be demo'ed in class. You are welcome to use the code from O'Rourke, Chapter 1 (downloadable in C or in Java from his website), which includes the convenient primitive `Diagonal`.